Shri Ram College of Commerce
Department of Mathematics

GE - 4 : Elements of Analysis, Semester IV (CBCS)

Assignment II - Unit - 1 : Sets, Functions & Sequences

MCQs and Short Answer Type Questions

MCQs

Q1) Let $g(x) = x^2$ and $f(x) = x+2$ for $x \in \mathbb{R}$. Then the composite function $h = gof$ is given by :

(a) x^2+2 (b) $x^2(x+2)$ (c) $(x+2)^2$ (d) $x+2x^2$

Q2) If A is a set with m elements and B is a set with n elements and if $A \cap B = \phi$, then $A \cup B$ has

(a) 0 elements (b) $m \times n$ elements (c) $m+n$ elements (d) $m-n$ elements

Q3) Which of the following sets is not denumerable

(a) $E_1 = \{2n : n \in \mathbb{N}\}$ (b) $E_2 = \{p : p \text{ is an even prime}\}$

(c) $E_3 = \{p : p \text{ is a prime number}\}$ (d) $E_4 = \{2^n : n \in \mathbb{N}\}$

Q4) Let S be a countable set. Which of the following statements is not true regarding the set S

(a) Every subset of S is countable (b) Every superset of S is countable

(c) There exists a surjection of $\mathbb{N}$ onto S (d) There exists an injection of S into $\mathbb{N}$.

Q5) If A and B are non-empty subsets of $\mathbb{R}$ that satisfy $a \leq b \ \forall\, a \in A$ and $\forall\, b \in B$, then which of the following statements is not true :

(a) $\inf A \geq \sup B$ (b) $\inf B \geq \sup A$ (c) $\sup A \leq \sup B$

(d) $\inf A \leq \inf B$

Q6) Let S be a non-empty bounded set in $\mathbb{R}$. Let $a > 0$ be a real number and let $aS = \{as : s \in S\}$. Then which of the following statements is always true

(a) $\inf(aS) = a \inf S$ (b) $\sup(aS) = a \inf S$

(c) $\inf(aS) = a \sup S$ (d) None of these

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Q7) Let S be a finite non-empty subset of $\mathbb{R}$. Which of the following statements is not true regarding S.

(a) S has a largest element.
(b) S has a smallest element.
(c) Sup S is a member of S.
(d) Inf S is not a member of S.

Q8) Which of the following statements is not true for $\mathbb{Q}$: the set of rational numbers:

(a) $\mathbb{Q}$ is an ordered field.
(b) $\mathbb{Q}$ is dense in $\mathbb{R}$.
(c) $\mathbb{Q}$ possesses the Completeness property
(d) $\mathbb{Q}$ is countable.

Q9) What is the smallest value of $n \in \mathbb{N}$ for which $b^n < \epsilon$, where $b = \cdot 5$ and $\epsilon = 10^{-3}$.

(a) 9 (b) 10 (c) 11 (d) 15

Q10) Which of the following statements is not true regarding the sequence $(0, 2, 0, 2, 0, 2, \ldots)$

(a) The sequence is bounded.
(b) The sequence is convergent.
(c) The sequence has a convergent subsequence.
(d) The sequence has a limit point.

Q11) Let $y_n = \sqrt{n+1} - \sqrt{n}$ $\forall n \in \mathbb{N}$. Which of the following statements is not true :

(a) $\{y_n\}$ converges
(b) $\{\sqrt{n}\, y_n\}$ converges
(c) $\{n y_n\}$ converges
(d) $\{y_n\}$ is a decreasing sequence.

Q12) Which of the following sequences is convergent

(a) $\langle 2^n \rangle$ (b) $\langle (\frac{1}{3})^n \rangle$ (c) $\langle (\frac{8}{7})^n \rangle$ (d) $\langle n^2 \rangle$

Q13) Which of the following sequences is not convergent

(a) $\{\frac{e^n}{3^n}\}$ (b) $\{(-1)^n \sqrt{n}\}$ (c) $\{\frac{\ln((e^4)^n)}{3n}\}$ (d) $\{\frac{(-1)^n}{\sqrt{n}}\}$

Q14) The sequence $\{\frac{n}{n+1}\}$ is

(a) Increasing (b) Unbounded (c) Decreasing (d) None of these

Q15) The value of $\lim_{n\to\infty} \frac{\log_5(n)}{\log_9(n)}$ is

(a) $\ln(9)/\ln(5)$ (b) $\ln(5)/\ln(9)$ (c) 5/9 (d) 9/5

Q16) The value of $\lim_{n\to\infty} \frac{\cos(n)}{\log_2(n)}$ is

(a) +1 (b) −1 (c) $+\infty$ (d) 0

Q17) Which of the following sequences is a Cauchy sequence

(a) $\{x_n\}$ defined by $x_1=1, x_2=2$ and $x_n = \frac{1}{2}(x_{n-1}+x_{n-2})$ $\forall n \geq 3$

(b) $\{x_n\}$ defined by $x_n = 1+\frac{1}{2}+\cdots+\frac{1}{n}$ $\forall n \in \mathbb{N}$

(c) $\{(-1)^n\}$ (d) $\{n+\frac{(-1)^n}{n}\}$

Q18) Let $x_1 = 8$ and $x_{n+1} = \frac{x_n}{2} + 2$ $\forall n \in \mathbb{N}$. Then which of the following statements is true regarding the sequence $\{x_n\}$

(a) Bounded above by 7 (b) $\{x_n\}$ is unbounded

(c) $\{x_n\}$ is an increasing sequence (d) $\{x_n\}$ is bounded below by 4

Q19) Given $0<a<1$ and $b>2$, which of the following sequences does not converge

(a) $\{a^n\}$ (b) $\{\frac{b^n}{2^n}\}$ (c) $\{\frac{n}{b^n}\}$ (d) $\{\frac{2^{3n}}{3^{2n}}\}$

Q20) The value of $\lim_{n\to\infty} (\frac{n^n}{n!})^{\frac{1}{n}}$ is

(a) e^2 (b) $\frac{1}{e^2}$ (c) e (d) $\frac{1}{e}$

Short Answer Type Questions :

Q1) Give an example of a convergent sequence $\{x_n\}$ of positive numbers with $\lim \frac{x_{n+1}}{x_n} = 1$

Q2) Give an example of a divergent sequence $\{x_n\}$ of positive numbers with $\lim \frac{x_{n+1}}{x_n} = 1$

Q3) Give an example of a convergent sequence $\{x_n\}$ of positive numbers with $\lim (x_n^{\frac{1}{n}}) = 1$

Q4) Give an example of a divergent sequence $\{x_n\}$ of positive numbers with $\lim (x_n^{\frac{1}{n}}) = 1$

Q5) Give examples of properly divergent sequences* $\{x_n\}$ and $\{y_n\}$ with $y_n \neq 0 \; \forall \; n \in \mathbb{N}$ such that

(a) $\left\{\frac{x_n}{y_n}\right\}$ is convergent (b) $\left\{\frac{x_n}{y_n}\right\}$ is properly divergent

Q6) Given an example of an unbounded sequence which has a properly divergent subsequence.

Q7) Give examples of a properly divergent sequence $\{x_n\}$ and a convergent sequence $\{y_n\}$ such that

(a) $\{x_n y_n\}$ is convergent (b) $\{x_n y_n\}$ is properly divergent

Q8) Give an example to show that the convergence of $\{|x_n|\}$ does not imply the convergence of $\{x_n\}$.

Q9) Let $A = (2, 5)$. Find an example of a sequence $\{x_n\}$ with $x_n \in A \; \forall \; n \in \mathbb{N}$ such that $\lim x_n = 5$

* Properly divergent Sequence : $\{x_n\}$ is said to be a properly divergent sequence if either $\lim x_n = \infty$ or $\lim x_n = -\infty$